1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 5,Module 4, Moment of Inertia Chapter 7, System of particles and Rotational Motion
Module Id	Keph_10704_eContent
Pre-requisites	Kinematics, laws of motion, basic vector algebra, center of mass, translatory motion, circular motion, rotation equations of rotational motion
Objectives	After going through this lesson, the learners will be able to: • **Define** Moment of inertia • **Distinguish between** mass and moment of inertia • **Derive** an expression for moment of inertia for a lamina about a vertical axis perpendicular to the plane of the lamina • **Deduce** S I Unit and dimensions of moment of inertia • **Understand** Radius of gyration • **Know** Perpendicular and Parallel axis theorems and their applications
Keywords	Moment of inertia, SI unit of moment of inertia, radius of gyration parallel axis theorem, perpendicular axis theorem

2. Development Team

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TABLE OF CONTENTS

1. UNIT SYLLABUS

Unit V: Motion of System of Particles and Rigid body

Chapter 7: System of particles and Rotational Motion

Centre of mass of a two-particle system; momentum conservation and centre of mass motion. Centre of mass of a rigid body; Centre of mass of a uniform rod.

Moment of a force; torque; angular momentum; law of conservation of angular momentum and its applications.

Equilibrium of rigid bodies; rigid body rotation and equations of rotational motion; comparison of linear and rotational motions.

Moment of inertia; radius of gyration; values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLBUS 8 Modules

The above unit has been divided into 8 modules for better understanding.

Module 1	• Rigid body • Centre of mass • Distribution of mass • Types of motion: Translatory, circulatory and rotatory
Module 2	• Centre of mass • Application of centre of mass to describe motion • Motion of centre of mass
Module 3	• Analogy of circular motion of a point particle about a point and different points on a rigid body about an axis • Relation v = rω • Kinematics of rotational motion • Equations for uniformly accelerated rotational motion
Module 4	• Moment of inertia • Difference between mass and moment of inertia • Derivation of value of moment of inertia for a lamina about a vertical axis perpendicular to the plane of the lamina • S I Unit • Radius of gyration • Perpendicular and Parallel axis theorems
Module 5	• Torque • Types of torque • Dynamics of rotational motion • $m = I\alpha$
Module 6	• Equilibrium of rigid bodies • Condition of net force and net torque • Applications
Module 7	• Law of conservation of angular momentum and its applications. • Applications
Module 8	• Rolling on plane surface • Horizontal • Inclined surface • Applications

Module 4

3. WORDS YOU MUST KNOW

- Rigid body: An object for which individual particles continue to be at the same separation over a period of time.
- Point object: If the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object.
- Frame of reference: Any reference frame the coordinates(x, y, z), which indicate the change in position of object with time
- Observer: Someone who is observing objects from any frame
- Rest: A body is said to be at rest if it does not change its position with surroundings with time
- Motion: A body is said to be in motion if it changes its position with respect to its surroundings with time
- Time elapsed: Time interval between any two observations of an object.
- Motion in one dimension: When the position of an object can be shown by change in any one coordinate out of the three (x, y ,z), also called motion in a straight line.
- Motion in two dimension: When the position of an object can be shown by changes any two coordinate out of the three (x, y,z), also called motion in a plane .
- Motion in three dimension: When the position of an object can be shown by changes in all three coordinate out of the three (x, y, z)
- Distance: The length of the path an object moves from its starting position to a final position SI unit m, this can be zero or positive.
- Displacement: The distance an object has moved from its starting position moves in a particular direction SI unit: m, this can be zero, positive or negative.
- Position vector: A vector representing the location of a point in space with respect to a fixed frame of reference

- Force: A push or a pull that can change the state of rest or motion of a body. It can also deform a body.
- Center of mass: The centre of mass of a system of particles moves as if all the mass of the system was concentrated at this centre of mass and all external forces were applied at that point
- Rectilinear motion: In rigid body motion along a straight path all point on the body move with the same speed and in the same direction.
- Curvilinear motion: In rigid body motion along a curved path where all points on the body travel in parallel curved tracks with the same speed.
- Circular motion: In a rigid body motion along a circular track the centre of mass of the body maintains a fixed distance from the center of the circular track.
- Kinetic Energy: Mechanical energy possessed by a body due to its mass and motion given by ½ $mv^{2.}$
- Angular displacements: For objects moving about a point or an axis displacement given by the line joining the point to the fixed point or the fixed axis
- Angular speed: Time rate of change of angular displacement is called angular velocity. Useful to describe body moving in a circular track or rotating about a fixed axis
- Linear speed: Time rate of change of position of an object
- Circular motion: When an object moving in a plane maintain a fixed distance from a fixed point at all times it is said to be in circular motion.

- Rotation: When an object moves about a fixed axis. The axis may or may not pass through the object

- Inertia: The property of a body to continue in its state of rest or motion unless an external unbalanced force acts on it

4. INTRODUCTION

In the unit of laws of motion we have studied about inertia as the property of any physical object to resist any change in its state of rest or motion; we termed it as inertia of rest

This includes changes to its speed, direction or state of rest.
One can also say, due to this property objects keep moving in a straight line at constant velocity.

Hence objects maintain their state of motion due to the **property of inertia and we refer to it as inertia of motion**

We are developing the study of rotational motion parallel to the study of translational motion with which we are familiar. We have yet to answer one major question in this connection.

What is the analogue of mass, measure of inertia in linear motion, in rotational motion? Will a body continue to state of rest or uniform rotational motion unless and until an unbalanced agency acts on it? Implies whether there is a Newton's first law of motion for rotational motion?

We shall attempt to answer this question in the present section.

5. ROTATIONAL KINETIC ENERGY AND MOMENT OF INERTIA

Let us try to get an expression for the kinetic energy of a rotating body. We know that for a rigid body rotating about a fixed axis with angular speed ω, each particle of the body moves in a circle.

Consider a point particle of mass m_i at a perpendicular distance r_i from the axis of rotation. The kinetic energy of the given particle is given as

$$K_i = \frac{1}{2}\ m_i\, v_i^2$$

where v_i is the speed of the given particle.

One can rewrite the above expression as speed of the particle is related with the angular speed by equation

$$v_i = r_i\ \omega_i$$

$$K_i = \frac{1}{2}\ m_i\, v_i^2 = \frac{1}{2}\ m_i\, r_i^2\, \omega_i^2$$

The total kinetic energy of the body can be written as the sum of the kinetic energies of all the particles constituting the given rotating body.

Therefore $$K = \sum_{i=1}^{n} K_i$$

$$K = \sum_{i=1}^{n} \frac{1}{2}\ m_i\, r_i^2\, \omega_i^2$$

As all the particles have same angular speed, hence

$$K = \frac{1}{2}\left(\sum_{i=1}^{n}\ m_i\, r_i^2\right)\omega^2$$

If one compares the above expression with the expression for kinetic energy in linear motion i.e

$$K = \frac{1}{2}\ m\, v^2$$

one can observe that quantity ω represent angular speed a rotational analogue of speed v, hence,

the quantity represented by

$$\sum_{i=1}^{n} m_i r_i^2$$

is a rotational analogue of mass.

The rotational analogue of mass is also called **moment of inertia of the given body about a chosen axis of rotation.**

THINK ABOUT THIS

- **It is the measure of inertia property for a given system in rotational motion.**
- **It determines the measure of opposition to the change of angular velocity just as mass in linear motion determines the measure of opposition to the change of linear velocity.**
- **It is represented as**

Moment of inertia $\quad I = \sum_{i=1}^{n} m_i r_i^2$

- **Here m_i is the mass of a point mass kept at a perpendicular distance r_i from the axis of rotation the equation (2) can be used when a body or system can be broken down into discrete point sized masses.**
- **For a fixed axis it can be considered to be a scalar quantity**
- **The SI unit of moment of inertia will be kg m^2.**
- **Its value depends on the axis of rotation and the distribution of mass about the given axis.**

- **One can write the expression of rotational kinetic energy, using equation given above as**

$$\mathbf{K=\frac{1}{2}\,I\,\omega^2}$$

Here I is the moment of inertia of a given body or system about the given axis and ω is the angular speed of the body or system about the given axis.

The property rotational inertia or moment of inertia is extremely important quantity, as it has large number of practical implications. The machines, auto engines and revolving train wheels, etc., that produce rotational motion; have a disc with a large moment of inertia, called a flywheel.

Because of its large moment of inertia, the flywheel resists the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for the passengers on the vehicle.

EXAMPLE

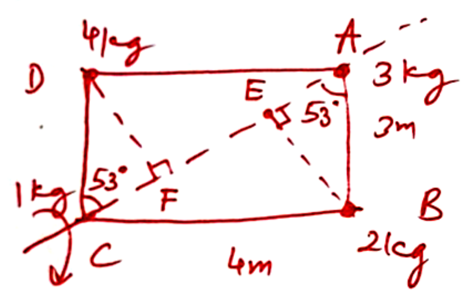
Four point masses lie at the corners of a rectangle with sides of length 3 m and 4 m as shown in the figure.

Find the moment of inertia about each diagonal.

SOLUTION

For each mass we need its perpendicular distance from the axis of rotation.

(i) Let's take first axis about diagonal AC.



The masses at vertex A and C will not contribute as they lie on the axis. The masses at vertices B and D are at a perpendicular distance, shown in figure, given as

$$BE = DF = 3\sin 53^{o} = 3 \times 4/5 = 2.4 \text{ m}$$

Therefore moment of inertia about axis AC is

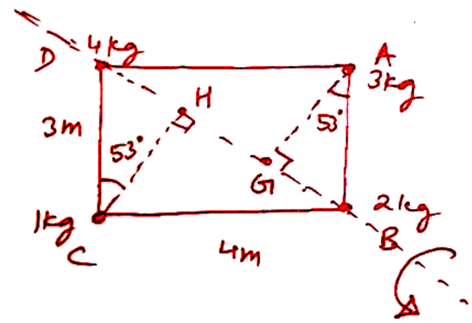
$$I = m_B\,(BE)^2 + m_B\,(DF)^2$$

$$I = 4 \times (2.4)^2 + 2 \times (2.4)^2 = 34.6 \text{ kg m}^2$$

(ii) Now axis about diagonal BD.

The masses at vertex B and D will not contribute as they lie on the axis. The masses at vertices C and A are at a perpendicular distance, shown in figure, given as:
CH= AG=3 sin53^o = 3 × 4/5 = 2.4 m

Therefore moment of inertia about axis AC is



$$I = m_C\,(CH)^2 + m_A\,(AG)^2$$

$$I = 1 \times (2.4)^2 + 3 \times (2.4)^2 = 23 \text{ Kg m}^2$$

6. MOMENT OF INERTIA FOR A CONTINUOUS DISTRIBUTION OF MASS

For a system of discrete point particles system the moment of inertia of the given system about a given axis is given as

$$I = \sum_{i=1}^{n} m_i\, r_i^2$$

where r_i is the perpendicular distance of particle of mass m_i from the axis of rotation.

In case system is a continuous distribution of mass then we divide given system into large no of infinitesimal elements of mass dm.

Let element moment of inertia be

$$dI = dm\ r^2\ .$$

The moment of inertia of the system can be determined by integrating over the given body as

$$I = \int dm\ r^2$$

The limits of the integral are according to orientation of the object with reference to the axis of rotation.

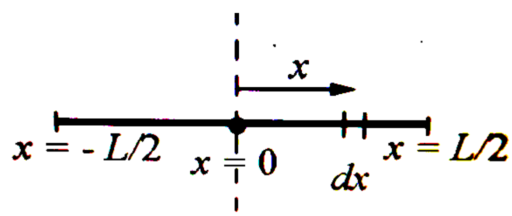
So if it is a rod, or a disc or any other geometrical shape one can calculate the moment of inertia

But say it is an irregular shaped lamina or solid, or irregular mass distribution solid we will have to consider

- **The axis of rotation ,**
- **The mass distribution about the axis of rotation**

Some of these examples may help you to understand a little better

EXAMPLE



A thin uniform rod of mass M and length L is mounted on an axis passing through the center of the rod, perpendicular to the plane of the rod. Calculate the moment of inertia about an axis that passes perpendicular to the rod through the center of mass of the rod.

SOLUTION:

The axis is through the center, so take the point O to be the center of mass of the rod. Choose the origin at the center of mass and the x-axis oriented along the rod, positive to the right in the figure. Let the length of a small element of the rod is denoted by

$$dL = dx \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Since the rod is uniform, the mass per unit length is a constant.

Therefore $$\lambda = \frac{\text{total mass}}{\text{total length}} = \frac{M}{L} = \frac{dm}{dL} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Hence, the mass in the infinitesimal length element as given in Equation (1), is given by

$$dm = \lambda\, dx = \frac{M}{L}\, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

When the rod rotates, the small element traces out a circle of radius x .i.e. the distance from the center of mass is the perpendicular distance from the axis. Therefore, moment of inertia of the element

$$dI = dm\ x^2 = = \frac{M}{L} x^2\, dx \qquad \text{(using value of dm from equation (2))}$$

Net moment of inertia of the rod can be written as

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} dI$$

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{M}{L} x^2\, dx = \frac{M}{L} \times \left(\frac{x^3}{3} \right)_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$I = \frac{M}{3\,L} \left[\frac{L^3}{8} - \frac{-L^3}{8} \right]$$

$$I = \frac{M}{3\,L} \times 2 \times \frac{L^3}{8}$$

$$\mathbf{I = \frac{M\ L^2}{12}}$$

Similarly one can find moment of inertia's of different objects.

The moment of inertia of a rigid body depends on:

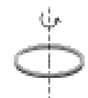
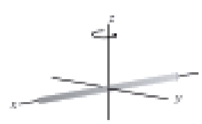
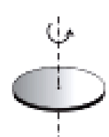
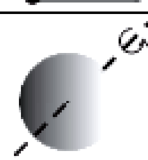
(i) **the mass of the body**

(ii) **its shape and size**

(iii) **distribution of mass about the axis of rotation**

(iv) **Position and orientation of the axis of rotation.**

Moment of Inertia for some objects of uniform distribution of mass has been given below

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		$M R^2$
(2)	Thin circular ring, radius R	Diameter		$M R^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$M L^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$M R^2/2$
(5)	Circular disc, radius R	Diameter		$M R^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		$M R^2$
(7)	Solid cylinder, radius R	Axis of cylinder		$M R^2/2$
(8)	Solid sphere, radius R	Diameter		$2 M R^2/5$

THINK ABOUT THIS

Some rotating installations are often seen by us, some pictures are displayed

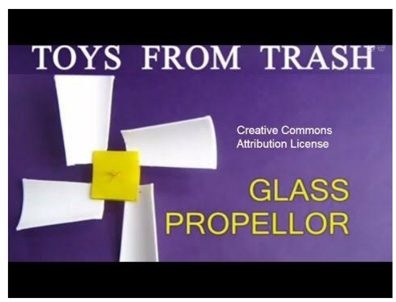
In each of the cases identify

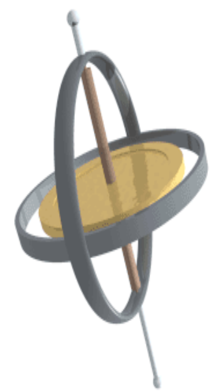
a) The axis of rotation

b) Distribution of mass about the axis of rotation

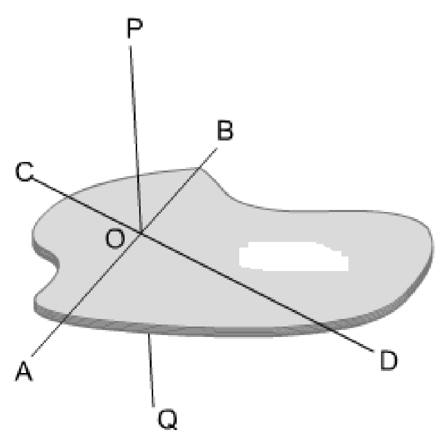
c) If a child sits in one of the seats would the moment of inertia change?

d) Can you calculate the moment of inertia of the merry go round, the gyroscope, a plastic cup windmill, or a top ? If yes, how will you do so?





7. THEOREM OF PERPENDICULAR AXIS



This theorem is applicable to bodies which are planar. In practice this means the theorem applies to flat bodies whose thickness is very small compared to their other dimensions (e.g. length, breadth or radius).

It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The figure shows a planar body or lamina.

An axis PQ perpendicular to the body through a point O is taken.

Two mutually perpendicular axes lying in the plane of the body and concurrent with PQ, i.e. passing through O, are taken as the AB and CD.

The theorem states that moment of inertia of the planar body is equal to the sum of moment of inertia of the body about axis AB and CD.

i.e. $\mathrm{I_{PQ}} = \mathrm{I_{AB}} + \mathrm{I_{CD}}$

Let us look at the usefulness of the theorem through an example.

EXAMPLE

What is the moment of inertia of a disc about one of its diameters, if the moment of inertia passing through its center and perpendicular to it is known?

SOLUTION

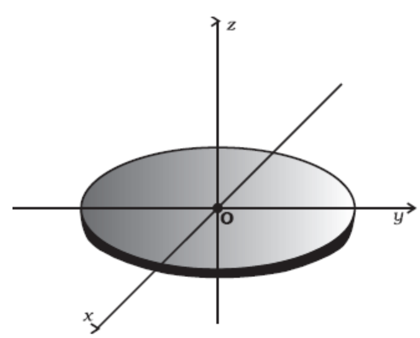
We know the moment of inertia of the disc about an axis perpendicular to it and through its centre

It is $\mathbf{I} = \frac{1}{2}\mathbf{MR^2}$

where M is the mass of the disc and R is its radius

The disc can be considered to be a planar body.

Hence the theorem of perpendicular axes is applicable to it. As shown in Figure, we take three concurrent axes through the centre of the disc, O as the x, y, z axes; x and y-axes lie in the plane of the disc and z is perpendicular to it.



By the theorem of perpendicular axes,

$$\mathrm{I_z} = \mathrm{I_x} + \mathrm{I_y}$$

Now, x and y axes are along two diameters of the disc, and by symmetry the moment of inertia of the disc is the same about any diameter.

Hence $I_x = I_y$ and $I_z = 2I_x$

But $I_z = \frac{1}{2}MR^2$

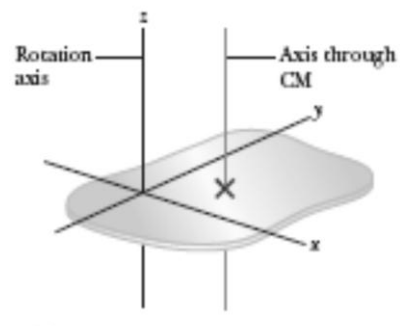
So finally, $I_x = \frac{1}{2} I_z = \frac{1}{4}MR^2$

Thus, the moment of inertia of a disc about any of its diameter is $\frac{1}{4}MR^2$.

8. THEOREM OF PARALLEL AXIS OR HUYGENS–STEINER THEOREM

This theorem is applicable to a body of any shape. It helps to determine the moment of inertia of a body about any axis, given the moment of inertia of the body about a parallel axis through the centre of mass of the body.

We shall only state this theorem and not give its proof.



We shall, however, apply it to a few simple situations which will be enough to convince us about the usefulness of the theorem. The theorem may be stated as follows:

The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

As shown in the Figure below, the z-axis passes through the centre of mass O of the rigid body and one more axis passes through the centre of mass of the given body.

Then according to the theorem of parallel axes, moment of inertia of the body about z axis is

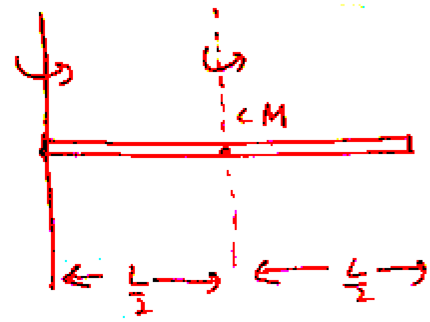
$$I = I_{CM} + Mh^2$$

where I and I_{CM} are the moments of inertia of the body about the z axis and axis through centre of mass respectively, M is the total mass of the body and h is the perpendicular distance between the two parallel axes.

EXAMPLE

What is the moment of inertia of a rod of mass M, length L about an axis perpendicular to it through one end?

SOLUTION



For the rod of mass M and length L, the moment of inertia about an axis passing through its centre is given as

$I = \frac{ML^2}{12}$.

Using the parallel axes theorem,

$I = I_{CM} + Mh^2$ with $h = \frac{L}{2}$, we get,

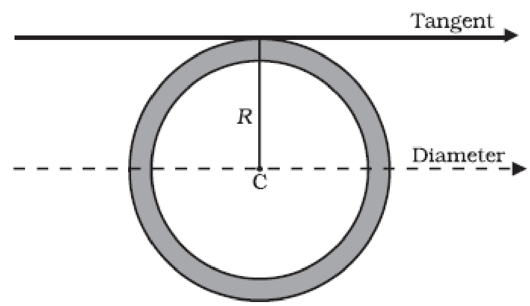
We can check this independently since I is half the moment of inertia of a rod of mass 2M and length 2L about its midpoint i.e.

$I = \frac{1}{2} \times$ moment of inertia of a rod of mass 2M and length 2L about an axis through its CM

$$\mathbf{I = \frac{1}{2} \times \frac{2\,M(2L)^2}{12} = \frac{ML^2}{3}}$$

EXAMPLE

What is the moment of inertia of a ring about a tangent to the circle of the ring?



SOLUTION

The tangent to the ring in the plane of the ring is parallel to one of the diameters of the ring. The distance between these two parallel axes is R, the radius of the ring. Using the parallel axes theorem, we can write

$$I_{\text{tangent}} = I_{\text{diameter}} + M\,R^2$$

I_{diameter} can be determined using perpendicular axis, as done for disc in solved problem2.

$$I_{\text{diameter}} = \frac{1}{2}M\,R^2$$

Therefore $\quad I_{\text{tangent}} = \frac{1}{2}M\,R^2 + M\,R^2 = \frac{3}{2}M\,R^2$

TRY THIS YOURSELF

i) **Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be 2*MR*^{2}/5, where *M* is the mass of the sphere and *R* is the radius of the sphere.**

ii) Given the moment of inertia of a disc of mass *M* and radius *R* about any of its diameters to be *MR*^{2} /4, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

iii) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

Show that the child's new kinetic energy of rotation is more than the initial

9. RADIUS OF GYRATION K

As the mass of a body resists a change in its state of linear motion, it is a measure of its inertia in linear motion. Similarly, as the moment of inertia about a given axis of rotation resists a change in its rotational motion, it can be regarded as a measure of rotational inertia of the body; it is a measure of the way in which different parts of the body are distributed at different distances from the axis.

Unlike the mass of a body, the moment of inertia is not a fixed quantity but depends on the orientation and position of the axis of rotation with respect to the body as a whole.

As a measure of the way in which the mass of a rotating rigid body is distributed with respect to the axis of rotation, we can define a new parameter, the radius of gyration.

It is related to the moment of inertia and the total mass of the body. Notice from the Table showing expression of moment of inertia of different objects about various axis of rotation, that in all cases, we can write

$I = Mk^2$, where k has the dimension of length.

For a rod, about the perpendicular axis at its midpoint,

i.e. $k^2 = \frac{L^2}{12}$

therefore, $k = \sqrt{\frac{L}{12}}$.

Similarly, k = R/2 for the circular disc about its diameter.

The length k is a geometric property of the body and axis of rotation. It is called the radius of gyration.

The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

Interestingly we can also say that the sum of kinetic energies of individual particles in the rotating system about the chosen axis of rotation , is the same as that of the entire mass at the radius of gyration about the axis of rotation

It has SI unit of m.

Can you think of an explanation for this?

For a rigid body let the moment of inertia about a given axis be

$I_1 = \sum_{i=1}^{n} m_i r_i^2$.

Let M be the mass of the body which can written as sum of masses of all the constituents comprising it.

Therefore $M = \sum_{i=1}^{n} m_i$.

If this whole mass is kept at a distance of k from the given axis of rotation then moment of inertia of this mass can be written as

$I_2 = M k^2$.

Using definition of radius of gyration

$$I_1 = I_2$$

$$M k^2 = \sum_{i=1}^{n} m_i r_i^2$$

hence $$k = \sqrt{\frac{\sum_{i=1}^{n} m_i r_i^2}{M}}$$

or one can also write it as

$$\text{Radius of gyration} = \sqrt{\frac{\text{moment of inertia of the system}}{\text{mass of the system}}}$$

Therefore, **the radius of gyration depends on the location of the axis of rotation and distribution of the mass about the axis of rotation.**

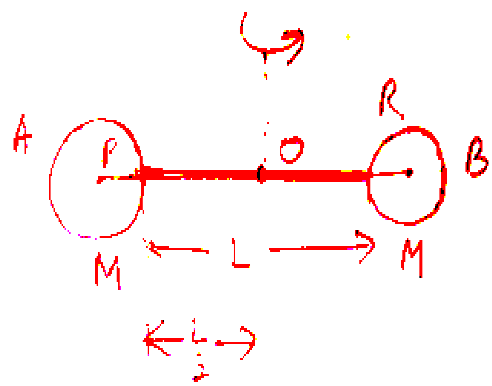
EXAMPLE

The dumbbell above consists of two homogenous, solid spheres, each of mass M and radius R. The spheres are connected by a thin, homogenous rod of mass m and length L. The entire dumbbell is rotating around the center of the rod. What is the moment of inertia of the dumbbell with respect to that axis?

SOLUTION

The moment of inertia of the given system can be written as

$$I = I_{sphere\ A} + I_{sphere\ B} + I_{rod} \ldots\ldots\ldots\ldots \quad (1)$$



The moment of inertia of the rod about an axis passing through its centre is

$$I_{rod} = \frac{1}{12}\, m\, L^2 \ldots\ldots\ldots \quad (2)$$

The moment of inertia of the solid sphere A can be determined using parallel axis theorem.

$$I = I_{CM} + Mh^2 \quad (3)$$

Here I_{CM} is the moment of inertia of the sphere A about its diameter .i.e.

$$I_{CM} = \frac{2}{5}\, M\, R^2 \quad (4)$$

And separation between the two parallel axes (axis passing through diameter of sphere A and axis passing through the centre of rod) is,

$$h = L + R \quad (5)$$

Therefore, the moment of inertia of sphere A about the axis passing through centre of rod is

$$I_{\text{sphere A}} = I_{CM} + Mh^2 = \frac{2}{5} M R^2 + M(L+R)^2 \ldots\ldots\ldots\ldots \quad (6)$$

The moment of inertia of the sphere B can determined in a similar manner, therefore

$$I_{\text{sphere B}} = \frac{2}{5} M R^2 + M(L+R)^2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

From equation (4), (5), (6) and (7), we can write

$$\mathbf{I = \frac{1}{12}\, m\, L^2 + \frac{4}{5}\, M\, R^2 + 2M(L+R)^2}$$

10. SUMMARY

- Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.
- The rotational analogue of mass is also called moment of inertia (MI) of the given body about given axis of rotation. Therefore, it is the measure of inertia property for a given system in rotational motion.
- Moment of inertia, determines the measure of opposition to the change of angular velocity just as mass in linear motion determines the measure of opposition to the change of linear velocity.
- For a system of point masses moment of inertia about a given axis is

  $I = \sum_{i=1}^{n} m_i r_i^2$

  Here m_i is the mass of a point mass kept at a perpendicular distance r_i from the axis of Rotation
- For a fixed axis MI can be considered to be a scalar quantity and has SI unit kg m^2.
- The moment of inertia of the system can be determined by integrating over the given body as

$$I = \int dm\ r^2$$

The limits of the integral are according to orientation of the object with reference to the axis of rotation.

- The moment of inertia of a rigid body depends on

(i) The mass of the body

(ii) Its shape and size

(iii) Distribution of mass about the axis of rotation

(iv) Position and orientation of the axis of rotation.

- Theorem of perpendicular axis

It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

- Theorem of parallel axis or Huygens–Steiner theorem

This theorem is applicable to a body of any shape. The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

- The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis. It has SI units of m.

Radius of gyration = $\sqrt{\frac{\text{moment of inertia of the system}}{\text{mass of the system}}}$

- The radius of gyration depends on the location of the axis of rotation and distribution of the mass about the axis of rotation.